

On the seismic age of the Sun

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Abstract. We use low-degree acoustic modes obtained by the BiSON to estimate the main-sequence age t_{\odot} of the Sun. The calibration is accomplished by linearizing the deviations from a standard solar model the seismic frequencies of which are close to those of the Sun. Formally, we obtain the preliminary value $t_{\odot} = 4.68 \pm 0.02$ Gy, coupled with an initial heavy-element abundance $Z = 0.0169 \pm 0.0005$. The quoted standard errors, which are not independent, are upper bounds implied under the assumption that the standard errors in the observed frequencies are independent.

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1. INTRODUCTION

Seismological calibration of stellar models against observed frequencies of low-degree modes was first discussed more than two decades ago (Christensen-Dalsgaard 1984, 1988; Ulrich 1986; Gough 1987), and can be regarded as a means of determining the main-sequence age of the Sun (Guenther 1989; Gough & Novotny 1990; Guenther & Demarque 1997; Weiss & Schlattl 1998; Dziembowski et al. 1999; Gough 2001; Bonanno, Schlattl & Paternò 2002). The procedure is to match certain appropriate seismic signatures of theoretical frequencies determined on a grid of stellar models with corresponding signatures obtained from the observations. The signatures are chosen to reflect principally the properties of the energy-generating core, where nuclear transmutation leaves behind an augmenting concentration of helium, lowering the sound speed relative to the environs and thereby providing a diagnostic of age. But the signatures are also susceptible to other properties of the stellar interior, which must be eliminated before a robust outcome can be achieved. For example, although the so-called small frequency separation is sensitive predominantly to the evolving stratification of the core, its dependence on the zero-age chemical abundances plays a significant contaminating role. Therefore it behoves us to seek an additional diagnostic to attempt to measure abundance separately. For given relative abundances of the heavy elements, the total absolute heavy-element abundance Z and the ${}^4\text{He}$ abundance Y are related by the requirement that the model has the observed luminosity and radius, principally the former. Therefore we need to aim at detecting only one of them. Here we use a signature indicative of the abrupt variation of the first adiabatic exponent γ_1 induced by the ionization of helium.

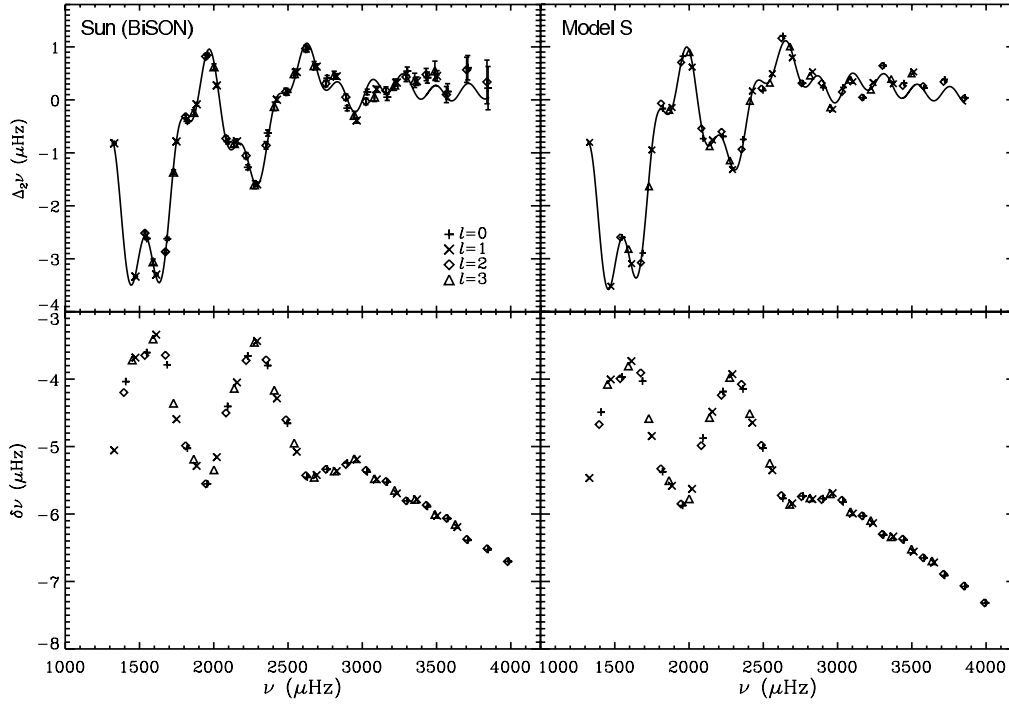


FIGURE 1. Top left: The symbols (with error bars obtained under the assumption that the raw frequency errors are independent) represent second differences, $\Delta_2\nu$, of low-degree solar frequencies from BiSON. Top right: The symbols are second differences $\Delta_2\nu$ of adiabatic pulsation eigenfrequencies of solar Model S. The solid curve in both panels is the diagnostic (2) – (5), whose eleven parameters have been adjusted to fit the data optimally. Bottom: The symbols denote contributions $\delta\nu$ to the frequencies produced by the acoustic glitches of the Sun (left panel) and Model S (right panel).

2. THE SEISMIC DIAGNOSTIC AND CALIBRATION METHOD

Any abrupt variation in the stratification of a star (relative to the scale of the inverse radial wavenumber of a seismic mode of oscillation), which here we call an acoustic glitch, induces an oscillatory component in the spacing of the cyclic eigenfrequencies $\nu_{n,l}$ of seismic modes, of order n and degree l . Our interest is principally in the glitch caused by the depression in the first adiabatic exponent $\gamma_1 = (\partial \ln p / \partial \ln \rho)_s$ (where p , ρ and s are pressure, density and specific entropy) caused by helium ionization. The deviation

$$\delta\nu := \nu - \nu_s \quad (1)$$

(from now on we omit the subscripts n, l) of the eigenfrequency from the corresponding frequency ν_s of a similar smoothly stratified star is the indicator of Y that we use in conjunction with the indicators of core structure to determine the main-sequence age.

Approximate expressions for the frequency contributions $\delta\nu$ arising from acoustic glitches in solar-type stars were recently presented by Houdek & Gough (2007). Here we improve them by adopting the appropriate Airy functions $\text{Ai}(-x)$ that are used as comparison functions in the JWKB approximations to the oscillation eigenfunctions. The complete expression for $\delta\nu$ is then given by

$$\delta\nu = \delta_\gamma\nu + \delta_c\nu, \quad (2)$$

	A	C	$-\delta\gamma_1/\gamma_1$
Sun (BiSON)	0.2764	1.785	0.04325
Model S	0.2780	1.818	0.04511

TABLE 1. Asymptotic fitting coefficients A , C (see equation 6) and $-\delta\gamma_1/\gamma_1 = A_{\Pi}/\sqrt{2\pi}\nu_0\Delta_{\Pi}$.

where

$$\begin{aligned} \delta_\gamma \nu &= -\sqrt{2\pi}A_{\Pi}\Delta_{\Pi}^{-1} \left[\nu + \frac{1}{2}(m+1)\nu_0 \right] \\ &\times \left[\mu\beta \int_0^T \kappa_{\text{I}}^{-1} e^{-(\tau-\eta\tau_{\Pi})^2/2\mu^2\Delta_{\Pi}^2} |x|^{1/2} |\text{Ai}(-x)|^2 d\tau \right. \\ &\quad \left. + \int_0^T \kappa_{\text{II}}^{-1} e^{-(\tau-\tau_{\Pi})^2/2\Delta_{\Pi}^2} |x|^{1/2} |\text{Ai}(-x)|^2 d\tau \right] \end{aligned} \quad (3)$$

arises from the variation in γ_1 induced by helium ionization, and

$$\begin{aligned} \delta_c \nu &\simeq A_c \nu_0^3 \nu^{-2} (1 + 1/16\pi^2 \tau_0^2 \nu^2)^{-1/2} \\ &\times \left\{ \cos[2\psi_c + \tan^{-1}(4\pi\tau_0\nu)] - (16\pi^2 \tilde{\tau}_c^2 \nu^2 + 1)^{1/2} \right\} \end{aligned} \quad (4)$$

results from the acoustic glitch at the base of the convection zone. Here, $m = 3.5$ is a constant, being a representative polytropic index in the expression for the approximate effective phase ψ appearing in the argument of the Airy function, and β , η and μ are constants of order unity which account for the relation between the acoustic glitches caused by the first and second stages of ionization of helium (Houdek & Gough 2007); τ is acoustic depth beneath the seismic surface of the star, and $T = 1/2\nu_0$ is the total acoustic radius of the star; τ_{Π} and Δ_{Π} are respectively the centre and the width of the He II acoustic glitch. The argument of the Airy function is $x = \text{sun}(\psi)|3\psi/2|^{2/3}$, where $\psi(\tau) = \kappa\omega\tilde{\tau} - (m+1)\cos^{-1}[(m+1)/\omega\tilde{\tau}]$ if $\tilde{\tau} > \tau_t$, and $\psi(\tau) = |\kappa|\omega\tilde{\tau} - (m+1)\ln[(m+1)/\omega\tilde{\tau} + |\kappa|]$ if $\tilde{\tau} \leq \tau_t$, in which $\tilde{\tau} = \tau + \omega^{-1}\epsilon_{\Pi}$, with $\omega = 2\pi\nu$, and τ_t is the location of the upper turning point; $\kappa(\tau) = [1 - (m+1)^2/\omega^2\tilde{\tau}^2]^{1/2}$, and $\kappa_{\text{I}} = \kappa(\eta\tau_{\Pi})$ and $\kappa_{\text{II}} = \kappa(\tau_{\Pi})$. Also $\psi_c = \kappa_c\omega\tilde{\tau}_c - (m+1)\cos^{-1}[(m+1)/\tilde{\tau}_c\omega] + \pi/4$, where $\kappa_c = \kappa(\tau_c)$ and $\tilde{\tau}_c = \tau_c + \omega^{-1}\epsilon_c$. The seven coefficients A_{Π} , Δ_{Π} , τ_{Π} , ϵ_{Π} , A_c , τ_c , ϵ_c are found by fitting the second difference

$$\Delta_2 \nu \equiv \nu_{n-1,l} - 2\nu_{n,l} + \nu_{n+1,l} \simeq \Delta_2(\delta_\gamma \nu + \delta_c \nu) + \sum_{k=0}^3 a_k \nu^{-k} \quad (5)$$

to the corresponding observations under the assumption that the errors in the frequency data are independent (see top panels of Figure 1). The last term in equation (5) approximates smooth contributions arising, in part, from wave refraction in the stellar core, from hydrogen ionization and from the superadiabaticity of the upper boundary layer of the convection zone, introducing four more fitting coefficients a_k ($k=0,\dots,3$).

The outcome of the fitting to the BiSON data (Basu et al. 2007) and to the adiabatically computed eigenfrequencies of solar Model S (Christensen-Dalsgaard et al. 1996) is displayed in Figure 1: the upper panels display the second differences, together with the fitted formula (5), the lower panels display the corresponding contributions $\delta\nu$ to the frequencies of oscillation from the acoustic glitches. To the resulting smooth (glitch free) frequencies ν_s , derived from equation (1), of both the solar observations (Basu et al.

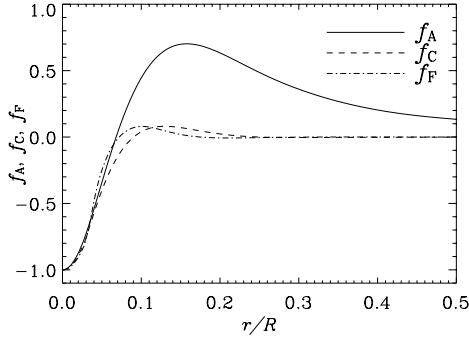


FIGURE 2. Functional forms f_X of the integrands ϕ_X in $X = \int_0^R \phi_X dr$, where $f_X(r) = \phi_X(r)/|\phi_X(0)|$ and where $X = A, C$, or F , plotted over the inner half of the interval $(0, R)$ of r . The parameters A, C and F are sensitive particularly to the structure of the core, being progressively more centrally concentrated. For the calibrations here we use only A and C (for F is more difficult to determine with confidence), whose values determined from the fit are listed in Table 1, together with the implied maximum depression $-\delta\gamma_1/\gamma_1 = A_{\Pi}/\sqrt{2\pi}\nu_0\Delta_{\Pi}$ in γ_1 caused by He II ionization.

2007) and the eigenfrequencies of the reference solar model (Model S) was fitted the asymptotic expression

$$\nu_{n,l} \sim (n + \frac{1}{2}l + \hat{\epsilon})\nu_0 - \frac{AL^2 - B}{\nu_{n,l}}\nu_0^2 - \frac{CL^4 - DL^2 + E}{\nu_{n,l}^3}\nu_0^4 - \frac{FL^6 - GL^4 + HL^2 - I}{\nu_{n,l}^5}\nu_0^6, \quad (6)$$

where $L^2 = l(l+1)$, from which we obtain the coefficients $\nu_0, \hat{\epsilon}, A, B, C, D, E, F, G, H$ and I , each of which is an integral of a function of the equilibrium stratification, some of which are displayed in Figure 2. The differences between the actual smoothed frequencies ν_s and the asymptotic expression (6) are plotted in Figure 3.

We have carried out age calibrations using combinations of the parameters

$$\xi_\alpha = (A, C, -\delta\gamma_1/\gamma_1), \quad \alpha = 1, 2, 3. \quad (7)$$

Presuming, as is normal, that Model S is parametrically close to the Sun, we consider the solar value ξ_α^\odot to be approximated by a two-term Taylor expansion of ξ_α about the value ξ_α^s for Models S:

$$\xi_\alpha^\odot = \xi_\alpha^s + \left(\frac{\partial \xi_\alpha}{\partial t_\odot}\right)_Z \Delta t_\odot + \left(\frac{\partial \xi_\alpha}{\partial Z}\right)_{t_\odot} \Delta Z - \epsilon_{\xi_\alpha}, \quad (8)$$

where Δt_\odot and ΔZ are the deviations of age t_\odot and initial heavy-element abundance Z from Model S, and ϵ_{ξ_α} are the formal errors in the calibration parameters. A (parametrically local) maximum-likelihood fit (again, assuming that the errors in the observed frequencies are independent) then leads to the following set of linear equations:

$$H_{\alpha j} C_{\alpha\beta}^{-1} H_{\beta k} \Theta_{0k} = H_{\alpha j} C_{\alpha\beta}^{-1} \Delta_{0\beta}, \quad (9)$$

in which $\Theta_k = (\Delta t_\odot, \Delta Z) + \epsilon_{\Theta k} = \Theta_{0k} + \epsilon_{\Theta k}$, $k = 1, 2$ is the solution vector subject to (correlated) errors $\epsilon_{\Theta k}$, $\Delta_\beta = \xi_\beta^\odot - \xi_\beta^s + \epsilon_{\xi\beta} = \Delta_{0\beta} + \epsilon_{\xi\beta}$, $C_{\alpha\beta}$ is the covariance matrix of the errors ϵ_{ξ_α} , and $H_{\alpha j} = [(\partial \xi_\alpha / \partial t)_Z, (\partial \xi_\alpha / \partial Z)_t]$, $j = 1, 2$. A similar set of equations is obtained for the formal errors $\epsilon_{\Theta k}$:

$$H_{\alpha j} C_{\alpha\beta}^{-1} H_{\beta k} \epsilon_{\Theta k} = H_{\alpha j} C_{\alpha\beta}^{-1} \epsilon_{\xi\beta}, \quad (10)$$

from which the error covariance matrix $C_{\Theta kq} = \overline{\epsilon_{\Theta k} \epsilon_{\Theta q}}$ can be computed with a Monte Carlo simulation.

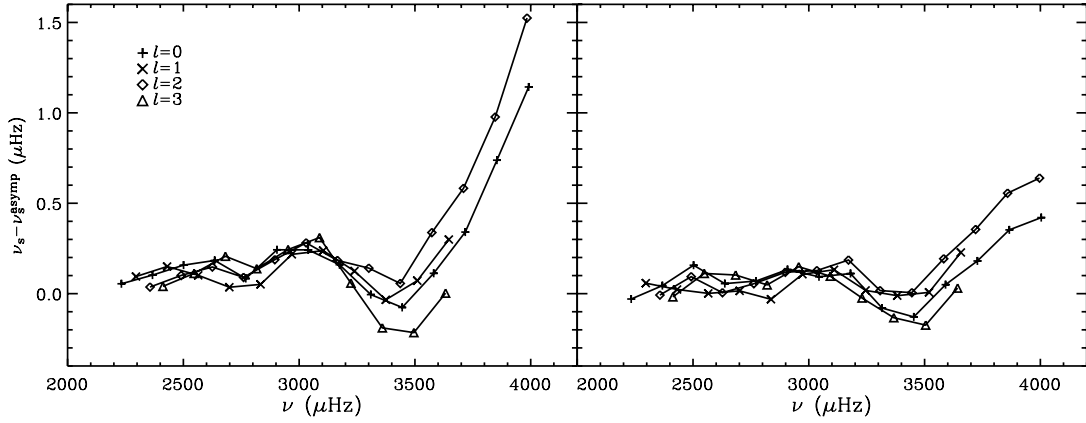


FIGURE 3. Differences between the smoothed frequencies ν_s of the Sun (left) and Model S (right), and the fitted asymptotic expression (6). Modes of like degree l are connected by solid lines.

TABLE 2. Partial derivatives $H_{\alpha j}$ obtained from two sets of calibrated evolutionary models for the Sun. Values with respect to age t_\odot are in units of Gy^{-1} .

$(\partial A / \partial t_\odot)_Z$	$(\partial A / \partial Z)_{t_\odot}$	$(\partial C / \partial t_\odot)_Z$	$(\partial C / \partial Z)_{t_\odot}$	$[\partial(-\delta\gamma_1/\gamma_1)/\partial t_\odot]_Z$	$[\partial(-\delta\gamma_1/\gamma_1)/\partial Z]_{t_\odot}$
-0.0469	-0.584	0.677	36.8	-0.00656	0.442

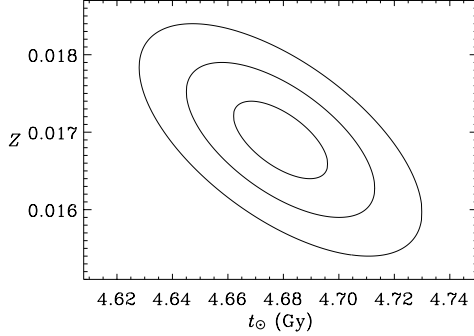
The partial derivatives $H_{\alpha j}$ are obtained from two sets of five calibrated evolutionary models for the Sun, computed with the evolutionary programme by Christensen-Dalsgaard (1982), and adopting the Livermore equation of state and the OPAL92 opacities. One set of models has a constant value for the heavy-element abundance $Z = 0.02$ but varying age; the other has constant age but varying Z . The values of the partial derivatives $H_{\alpha j}$ are listed in Table 2.

3. RESULTS

Age calibrations using the different combinations of the parameters ξ_α are summarized in Table 3; error contours associated with the first entry are plotted in Figure 4. In all cases the age found is greater than currently accepted values. The values of Z should not be regarded strictly as statements about the initial heavy-element abundance, but rather as measures of the opacity in the radiative interior. Asplund et al. (2004) have argued that the photospheric abundances of C, N and O had previously been overestimated, suggesting that the actual total heavy-element abundance is rather lower than previously believed. However, that cannot imply that the opacity in the solar interior is necessarily comparably lower because it has been implicitly calibrated here (by accepting the tenets of solar-evolution theory, and the OPAL opacity calculations upon which the models are based), and indeed the opacity has already been determined seismologically from a broader spectrum of modes than has been adopted here (Gough 2004). The matter raised by Asplund et al. therefore challenges either the opacity calculations, the nuclear reaction rates, or the basic physics of stellar evolution, not helioseismology, as some spectators have surmised. As we know already from seismological structure inversions, the solar models are not accurate by helioseismological standards. Therefore the prop-

TABLE 3. Age calibrations with different combinations of ξ_α .

ξ_α	t_\odot (Gy)	$C_{\Theta 11}^{1/2}$	Z	$C_{\Theta 22}^{1/2}$	$-(-C_{\Theta 12})^{1/2}$
$A, C, -\delta\gamma_1/\gamma_1$	4.679	0.017	0.0169	0.0005	-0.0023
A, C	4.658	0.023	0.0177	0.0007	-0.0037
$A, -\delta\gamma_1/\gamma_1$	4.673	0.017	0.0165	0.0007	-0.0019
$C, -\delta\gamma_1/\gamma_1$	4.700	0.028	0.0169	0.0005	-0.0029

**FIGURE 4.** Error ellipses for the calibration using all three parameters ξ_α : solutions (t_\odot, Z) satisfying the frequency data within 1, 2 and 3 standard errors in those data reside in the inner, intermediate and outer ellipses, respectively.

erties inferred from these calibrations could be more contaminated by systematic error than by errors in the observed frequencies.

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